Detection of de novo copy number alterations in case-parent trios using the R package MinimumDistance

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Abstract

For the analysis of case-parent trio genotyping arrays, copy number variants (CNV) appearing in the offspring that differ from the parental copy numbers are often of interest (de novo CNV). This package defines a statistic, referred to as the minimum distance, for identifying de novo copy number alterations in the offspring. We smooth the minimum distance using the circular binary segmentation algorithm implemented in the Bioconductor package DNAcopy. Trio copy number states are inferred from the maximum a posteriori probability of the segmented data, incorporating information from the log R ratios and B allele frequencies. As both log R ratios and B allele frequencies are estimable from Illumina and Affymetrix arrays, this package supports de novo copy number inference in both platforms.

1 Introduction

There are numerous R packages available from Bioconductor for smoothing copy number alterations. For example, the biocview CopyNumberVariants in the 2.9 release of Bioconductor lists 27 packages. For the analysis of germline diseases, hidden Markov models have emerged as a popular tool as inference regarding the latent copy number state incorporates information from both the estimates of copy number (or relative copy number) and the allelic frequencies, such as the B allele frequency [4] or genotype calls [1, 7, 5]. For the analysis of somatic cell diseases such as cancer, algorithms that segment the genome into regions of constant copy number (referred to here as segmentation algorithms) may be more preferable for the detection of copy number aberrations (CNA) as a mixture of cells with different copy numbers give rise to mosaic (non-integer) copy numbers. Examples include circular binary segmentation implemented in the R package DNAcopy and the GLAD, both of which were originally developed for array CGH platforms [3, 2, 6]. One disadvantage of segmentation algorithms is that inference regarding duplication and deletions is not directly available.

More recently, HMMs and segmentation algorithms have been developed for inferring common regions of copy number alterations in multiple samples. However, relatively few algorithms are available for inferring copy number alterations, especially de novo copy number alterations, in family-based study designs involving case-parent trios. Instead, a common strategy has been a two-step approach of identifying copy number alterations in the individual samples and then comparing the results across samples to infer whether an alteration observed in the offspring is de novo. A disadvantage of the two-step approach is that unadjusted sources of technical variation, such as waves, can contribute to false positives. The joint HMM implemented in the PennCNV software is one of the few algorithms to date that provides direct inference regarding de novo alterations in case parent study designs.

This package develops an alternative framework for inferring regions of de novo copy number alterations in case-parent trios. Like the PennCNV joint HMM, inference regarding de novo alterations integrates information from both the log R ratios and B allele frequencies. Differences in the two approaches are most apparent in non-HapMap experimental datasets in which technical and experimental sources of variation appear to contribute to a large number of false positives. This vignette describes the analysis pipeline from preprocessed and normalized estimates of copy number and allele frequencies to inference of de novo copy number alterations. The workflow is illustrated using a publicly available HapMap trio and a publicly available oral cleft trio, each assayed on the Illumina 610quad array.

2 Data input

```
R> library(oligoClasses)
R> library(VanillaICE)
R> library(SummarizedExperiment)
R> library(MinimumDistance)
R> foreach::registerDoSEQ()
```

2.1 Reading and organizing the annotation on the markers

We require that marker-level annotation is represented as a *GRanges*-derived class. To read the plain text annotation file, we use the **fread** function provided in the **data.table** package. In addition, we define an indicator for whether the marker is polymorphic using the 'Intensity Only' flag. The *SnpGRanges* class created from the **fgr** object in the following code-chunk ensures that this binary indicator is created and can be reliably accessed.

Ideally, one should include the genome build and the chromosome lengths appropriate to the build. Here, we extract the metadata on the chromosomes using the BSgenome Bioconductor package for the hg18 build. Finally, we sort the fgr object such that the chromosomes are ordered by their seqlevels and the markers are ordered by their genomic position along the chromosome.

```
R> library(BSgenome.Hsapiens.UCSC.hg18)
R> sl <- seqlevels(BSgenome.Hsapiens.UCSC.hg18)
R> seqlevels(fgr) <- sl[sl %in% seqlevels(fgr)]
R> seqinfo(fgr) <- seqinfo(BSgenome.Hsapiens.UCSC.hg18)[seqlevels(fgr),]
R> fgr <- sort(fgr)</pre>
```

2.2 Organizing the marker-level summaries

The abbreviated plain text files included with this package that contain the log R ratios and B allele frequencies from 2 trios are listed below.

```
R> files <- list.files(extdir, full.names=TRUE, recursive=TRUE, pattern="FinalReport")
```

We parse these files into a specific format such that all downstream steps for copy number estimation no longer depend on the format of the source files. At this point, we encapsulate the names of the source files ('sourcePaths'), the location of where we intend to store the parsed data ('parsedPath'), and the genomic marker annnotation created in the preceding section ('rowRanges') in a single object called an *ArrayViews*.

```
R> ##
R> ## Where to keep parsed files
R> ##
R> parsedDir <- "ParsedFiles"
R> if(!file.exists(parsedDir)) dir.create(parsedDir)
R> views <- ArrayViews(rowRanges=fgr, sourcePaths=files, parsedPath=parsedDir)
R> show(views)
```

```
class 'ArrayViews'
  No. files : 6
  No. markers: 11780
```

Because the format of the source files depends on upstream software, we read one file and store information regarding its parsing so that subsequent files can be parsed in a similar fashion. We use the fread to read in the first file.

```
R> ## read the first file
R> dat <- fread(files[1], skip="[Data]")</pre>
R> head(dat, n=3)
      SNP Name Allele1 - AB Allele2 - AB B Allele Freq
1: cnvi0005126
                                                   0.0085
2: cnvi0005127
                                                   0.0081
3: cnvi0005128
                                                   0.0063
   Log R Ratio
       -0.7214
1:
2:
       -0.8589
3:
       -0.1640
```

Next, we select which columns we plan to keep. Again, the required data for downstream processing is the name of the SNP identifier, the log R ratios, and B allele frequencies.

We also specify the order in which we will store the marker-level summaries by matching the rownames of the views object with the names of the markers in the source file:

```
R> index_genome <- match(names(fgr), dat[["SNP Name"]])</pre>
```

Similar to the parameter classes defined in Rsamtools, we encapsulate the information for parsing the columns and rows of the source files in a class. In addition, we specify which variable names in the source file refers to log R ratios ('cnvar'), B allele frequences ('bafvar'), and genotypes ('gtvar').

The parseSourceFile will parse a single file in the views object (by default, the first file) according to the parameters for reading the data in the scan_params object and write the output to the parsedPath directory. In particular, the parseSourceFile returns NULL.

```
R> parsedPath(views)
[1] "ParsedFiles"
R> parseSourceFile(views[, 1], scan_params)
```

To apply the function parseSourceFile to all arrays in the views object, one can use the functional sapply.

```
R> invisible(sapply(views, parseSourceFile, param=scan_params))
```

Apart from confirming their existance, the user should not have a need to directly access the parsed files. Utilities for querying these files are provided through the views object.

```
R> head(list.files(parsedPath(views)), n=3)
[1] "FinalReport1664_baf.rds" "FinalReport1664_gt.rds"
[3] "FinalReport1664_lrr.rds"
```

[5] Finalkepolt1004_111.1ds

2.3 Accessors for the parsed data

The preference for writing the parsed data to disk rather than keeping the data in RAM is simply that the latter does not scale to projects involving thousands of samples. For the former, slices of the parsed data easily be accessed from the parsedPath directory via methods defined for the ArrayViews class. For example, one can use accessors for the low-level summaries directly: 1rr, baf, and genotypes for log R ratios, B allele frequencies, and genotypes, respectively. The user has the option of either subsetting the views object or subsetting the matrix returned by the accessor to extract the appropriate data slice. In the following examples, we access data on the first 2 markers and sample indices 2-4.

```
R> lrr(views)[1:2, 2:4]
           FinalReport1675.txt FinalReport1686.txt
rs12789205
                         -0.169
                                              0.191
                         -0.182
                                              -0.114
rs2114088
           FinalReport6841.txt
                         -0.084
rs12789205
rs2114088
                          0.253
R> ## or
R> lrr(views[1:2, 2:4])
           FinalReport1675.txt FinalReport1686.txt
rs12789205
                         -0.169
                                              0.191
rs2114088
                         -0.182
                                              -0.114
           FinalReport6841.txt
rs12789205
                         -0.084
rs2114088
                          0.253
R> ## B allele frequencies
R> baf(views[1:2, 2:4])
           FinalReport1675.txt FinalReport1686.txt
rs12789205
                          0.529
                                               0.549
                                               0.000
rs2114088
                          0.000
           FinalReport6841.txt
rs12789205
                          0.481
rs2114088
                          0.000
R> ## potentially masked by function of the same name in crlmm
R> VanillaICE::genotypes(views)[1:2, 2:4]
           FinalReport1675.txt FinalReport1686.txt
rs12789205
                              2
                                                   2
rs2114088
                              1
                                                   1
           FinalReport6841.txt
rs12789205
                              2
rs2114088
                              1
```

More often, it is useful to extract the low-level data in a RangedSummarizedExperiment-derived class such that meta-data on the samples remains bound to the columns of the assay data (log R ratios / B allele frequencies) and meta-data on the rows remains bound to the rows of the assay data. This is accomplished by applying the MinDistExperiment function to a views object, as described in the following section.

3 Binding the genomic, marker, and individual-level data

3.1 Pedigree annotation

The container for a single pedigree is the ParentOffspring class.

For families with multiple affected offspring, the argument to offspring can be a character-vector with length greater than 1. We can store any number of *ParentOffspring* objects in a list using the *ParentOffspringList* class. In particular, for the 2 trios provided in the VanillalCE package

For a single pedigree, the *MinDistExperiment* encapsulates the annotation on the pedigree, the genomic annotation of the markers, and the marker-level summary statistics. Note, however, that the sample identifiers in the pedigree are not the same as the file identifiers used by default to create the R object views. To resolve the naming issue, we read in separate file provided in the VanillalCE package:

```
R> sample_info <- read.csv(file.path(extdir, "sample_data.csv"), stringsAsFactors=FALSE)
R> ind_id <- setNames(gsub(" ", "", sample_info$IndividualID), sample_info$File)
R> colnames(views) <- ind_id[gsub(".txt", "", colnames(views))]</pre>
```

3.2 MinDistExperiment class

The constructor function for MinDistExperiment extracts from the views object only the data relevant for the provided pedigree.

```
R> me <- MinDistExperiment(views, pedigree=ped_list[[2]])
R> colnames(me)
[1] "22169_03" "22169_02" "22169_01"
R> me
class: MinDistExperiment
dim: 11780 3
metadata(0):
assays(2): cn baf
rownames(11780): rs12789205 rs2114088 ... rs5994329
    rs1055232
rowData names(1): isSnp
colnames(3): 22169_03 22169_02 22169_01
colData names(0):
```

4 Detection of de novo copy number variants

A container for the various parameters used to segment and call de novo copy number variants is provided by the *MinDistParam*. A constructor function of the same name provides a default parametrization that works well in most instances:

```
R> params <- MinDistParam()
R> show(params)

An object of class 'MinDistParam'
   call segments with |seg.mean|/MAD > 0.75
   Setting nMAD() to smaller values will increase the number of segments that are called.
   DNAcopy settings:
    alpha: 0.01
    min.width: 2
   undo.splits: none
   undo.SD: 3
   See segment() for description of DNAcopy parameters
```

The parameters in this class are organized according to function. For example, the argument dnacopy takes an argument of class DNAcopyParam that contains setting that are passed to the segment function for the implementation of circular binary segmentation in the R package DNAcopy. Changing the parameters for the segmentation is most easily accomplished by a call to the constructor. E.g.,

```
R> segment_params <- DNAcopyParam(alpha=0.01)
R> params <- MinDistParam(dnacopy=segment_params)</pre>
```

Several of the parameters relate to a priori assumptions of the probability of a non-Mendelian transmission. These a priori assumptions are defined in PennCNV [7] and encapsulated in the *PennParam* class.

```
R> penn_param <- PennParam()
R> show(penn_param)

Object of class `PennParam'
    121    trio states
    state names: 000,100,200,300,400,010 ...
    reference state: 222
    PennCNV Table 1: 121    vector
    PennCNV Table 3: 5 x 5 x 5 x 5 x 5 x 5 array
    probability non-Mendelian: 1.5e-06
    initial state probabilities: 0.2,0.2,0.2,0.2,0.2
    transition prob: 5 x 5 matrix
    See table1(), table3(), state(), stateNames(), referenceState()
```

Finally, parameters for the emission probabilities computed by the R package VanillalCE are encapsulated in the *EmissionParam* class. Again, default values for the emission probabilities will be created automatically as part of the params object if none is specified.

Segmentation and posterior calls. For a given trio, the signed minimum absolute difference of the offspring and parental $\log_2 R$ ratios (r) is defined as

$$d \equiv (r_{\mathrm{O}} - r_{\mathrm{M}}) \times \mathbb{I}_{[|r_{\mathrm{O}} - r_{\mathrm{E}}| > |r_{\mathrm{O}} - r_{\mathrm{M}}|]} + (r_{\mathrm{O}} - r_{\mathrm{E}}) \times \mathbb{I}_{[|r_{\mathrm{O}} - r_{\mathrm{E}}| < |r_{\mathrm{O}} - r_{\mathrm{M}}|]}. \tag{1}$$

If the offspring copy number changes within a minimum distance segment (as determined by the segmentation of the offspring copy number), the start and stop position of the minimum distance segments may be edited. The approach currently implemented is to define a new start and stop position if a breakpoint for the offspring segmentation occurs in the minimum distance interval. To illustrate, the following diagram uses vertical dashes (|) to denote breakpoints:

In the above illustration, posterior calls are provided for the 3 segments indicated in line 3 instead of the single segment in line 1. Two additional steps are therefore required: (1) segmentation of the offspring log R ratios and (2) editing of the minimum distance breakpoints when appropriate. The following codechunk, segments the log R ratios for all chromosomes in the me object and edits the ranges for the minimum distance when conflicts with the offspring segmentation boundaries arise (as illustrated above).

```
R> mdgr <- segment2(me, params)
Analyzing: X22169_03
Analyzing: X22169_02
Analyzing: X22169_01
Analyzing: md_22169_01</pre>
```

For each minimum distance segment in which the mean minimum distance is above a user-specificed cutoff in absolute value (specified in terms of the median absolute deviations of the minimum distance), a trio copy number state is assigned from the maximum a posteriori estimate.

```
R> ## the threshold in terms of the number of median absolute deviations from zero R> nMAD(params) R> md_g \leftarrow MAP2(me, mdgr, params) R> show(md_g)
```

5 Inspecting, Filtering, and plotting de novo CNV inference

5.1 Filtering

There are several meta-data columns stored in the *GRanges*-derived summary useful for filtering the set of genomic intervals. All the available parameters for filtering the fit object are stored in the parameter class *FilterParam* of which an instance can be created by its constructor of the same name without any arguments.

```
R> filter_param <- FilterParamMD()
R> show(filter_param)

An object of class 'FilterParamMD'
  min. posterior probability of trio CNV call: 0.99
  min. no. of markers spanned by segment : 10
  min. width of segment : 1
  121 selected trio CN states : 000, 100, 200, 300, 400, 010, ...
  selected seqnames : chr1, chr2, chr3, chr4, chr5, chr6 ...
```

To apply the default filter parameters to the fit object, we use the cnvFilter function. The cnvFilter function returns only the set of genomic ranges satisfying the filters. For example,

```
[2]
       chr22 17281004-17633332
                                    * | md 22169 01
                                                       -0.3178
[3]
       chr22 17670969-18670691
                                    * | md_22169_01
                                                       -0.3540
[4]
       chr22 19066315-19792353
                                    * | md_22169_01
                                                       -0.3359
       log_RR log_odds prob_MAP prob_222 prob_221
    <numeric> <numeric> <numeric> <numeric> <numeric>
      72.737
[1]
                -82.214
                               1
                                          0
[2]
     146.106
                    Inf
                                1
                                          0
[3]
     697.008
                    Inf
                                          0
                                1
                                                     1
Γ47
     430.425
                    Inf
                                1
          calls number_probes
    <character>
                    <integer>
[1]
            302
                           25
                           88
[2]
            221
            221
                          278
[3]
[4]
            221
                          208
```

seqinfo: 3 sequences from hg18 genome

The helper functions denovoHemizygous and denovoHomozygous create commonly used filters.

R> denovoHemizygous(md_g)

```
MDRanges object with 3 ranges and 9 metadata columns:
      segnames
                          ranges strand
                                               sample seg.mean
         <Rle>
                       <IRanges> <Rle> | <character> <numeric>
  [1]
         chr22 17281004-17633332
                                      * | md_22169_01
                                                        -0.3178
  [2]
         chr22 17670969-18670691
                                      * | md_22169_01
                                                        -0.3540
         chr22 19066315-19792353
  [3]
                                      * | md_22169_01
                                                         -0.3359
         log_RR log_odds prob_MAP prob_222 prob_221
      <numeric> <numeric> <numeric> <numeric> <numeric>
       146.106
  [1]
                      Inf
                                  1
  [2]
        697.008
                      Inf
                                  1
                                            0
                                                       1
  [3]
        430.425
                      Inf
                                  1
                                            0
                                                       1
            calls number_probes
      <character>
                      <integer>
  [1]
              221
                             88
              221
  [2]
                            278
  [3]
              221
                            208
```

seqinfo: 3 sequences from hg18 genome

R> denovoHomozygous(md_g)

sequino. S sequences from figro genome

Equivalently, one could customize the filter parameteters to acheive the same result:

```
R> select_cnv <- FilterParamMD(state=c("220", "221", "223"), seqnames="chr22")
R> cnvs <- cnvFilter(md_g, select_cnv)</pre>
R> cnvs
MDRanges object with 3 ranges and 9 metadata columns:
      seqnames
                           ranges strand
                                                 sample
                                                         seg.mean
                        <!Ranges>
         <Rle>
                                           <character> <numeric>
                                   <Rle>
  [1]
         chr22 17281004-17633332
                                           md_22169_01
                                                          -0.3178
  [2]
         chr22 17670969-18670691
                                        * | md_22169_01
                                                          -0.3540
  [3]
         chr22 19066315-19792353
                                       * | md_22169_01
                                                          -0.3359
         log_RR log_odds prob_MAP prob_222 prob_221
      <numeric> <numeric> <numeric> <numeric> <numeric>
        146.106
  [1]
                       Inf
                                   1
  [2]
        697.008
                       Inf
                                   1
                                              0
                                              0
  [3]
        430,425
                                   1
                                                        1
                       Inf
            calls number_probes
      <character>
                       <integer>
  [1]
              221
                              88
  [2]
              221
                             278
  [3]
              221
                             208
```

seqinfo: 3 sequences from hg18 genome

5.2 Visualization

This produces a somewhat unsatisfactory result in that there are 2 additional denovo hemizygous deletions nearby that likely comprise one denovo segment.

R> denovoHemizygous(md_g)

MDRanges object with 3 ranges and 9 metadata columns: seqnames ranges strand sample seg.mean <Rle> <IRanges> <Rle> | <character> <numeric> [1] chr22 17281004-17633332 * | md_22169_01 -0.3178 [2] chr22 17670969-18670691 * | md_22169_01 -0.3540 [3] chr22 19066315-19792353 * | md_22169_01 -0.3359 log_RR log_odds prob_MAP prob_222 prob_221 <numeric> <numeric> <numeric> <numeric> <numeric> Γ17 146.106 Inf 1 0 [2] 697.008 Inf 1 0 1 [3] 430.425 Inf 1 1 calls number_probes <character> <integer> [1] 221 88 [2] 221 278 221 208 [3]

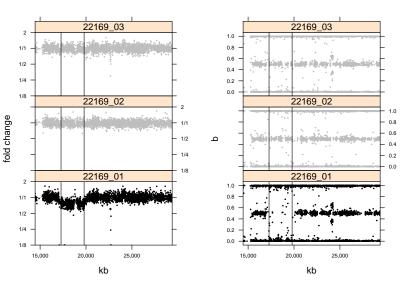
seqinfo: 3 sequences from hg18 genome

In the following code-chunk, we reduce the denovo hemizygous deletions and update the posterior probabilities for the MAP estimates. We use a combination of lattice and grid to visualize the marker-level summaries and copy number inference for the trio.

```
R> library(grid)
R> g2 <- reduce(denovoHemizygous(md_g), min.gapwidth=500e3)</pre>
```

```
R> post <- MAP2(me, g2, params)
R> g2 <- denovoHemizygous(post)
R> vps <- pedigreeViewports()
R> grid.params <- HmmTrellisParam()
R> p <- plotDenovo(me, g2, grid.params)
R> pedigreeGrid(g=g2, vps=vps, figs=p)
```





size (kb) :2,511.3 fold change :0.8 MAP estimate :221 posterior Pr :1 posterior odds :221 or 220 221 or 220 :Inf posterior RR :4.5e+111

6 Acknowledgements

Moiz Bootwalla contributed to early versions of this vignette.

7 Session information

R> toLatex(sessionInfo())

- R version 4.2.1 (2022-06-23), x86_64-pc-linux-gnu
- Locale: LC_CTYPE=en_US.UTF-8, LC_NUMERIC=C, LC_TIME=en_GB, LC_COLLATE=C, LC_MONETARY=en_US.UTF-8, LC_MESSAGES=en_US.UTF-8, LC_PAPER=en_US.UTF-8, LC_NAME=C, LC_ADDRESS=C, LC_TELEPHONE=C, LC_MEASUREMENT=en_US.UTF-8, LC_IDENTIFICATION=C
- Running under: Ubuntu 20.04.5 LTS
- Matrix products: default
- BLAS: /home/biocbuild/bbs-3.16-bioc/R/lib/libRblas.so
- LAPACK: /home/biocbuild/bbs-3.16-bioc/R/lib/libRlapack.so
- Base packages: base, datasets, grDevices, graphics, grid, methods, stats, stats4, utils
- Other packages: BSgenome 1.66.0, BSgenome.Hsapiens.UCSC.hg18 1.3.1000, Biobase 2.58.0, BiocGenerics 0.44.0, Biostrings 2.66.0, GenomeInfoDb 1.34.0, GenomicRanges 1.50.0, IRanges 2.32.0, MatrixGenerics 1.10.0, MinimumDistance 1.42.0, S4Vectors 0.36.0, SummarizedExperiment 1.28.0,

- VanillaICE 1.60.0, XVector 0.38.0, data table 1.14.4, matrixStats 0.62.0, oligoClasses 1.60.0, rtracklayer 1.58.0
- Loaded via a namespace (and not attached): BiocIO 1.8.0, BiocManager 1.30.19, BiocParallel 1.32.0, DBI 1.1.3, DNAcopy 1.72.0, DelayedArray 0.24.0, GenomeInfoDbData 1.2.9, GenomicAlignments 1.34.0, Matrix 1.5-1, RCurl 1.98-1.9, Rcpp 1.0.9, RcppEigen 0.3.3.9.2, Rsamtools 2.14.0, VGAM 1.1-7, XML 3.99-0.12, affyio 1.68.0, askpass 1.1, base64 2.0.1, beanplot 1.3.1, bit 4.0.4, bitops 1.0-7, codetools 0.2-18, compiler 4.2.1, crayon 1.5.2, crlmm 1.56.0, ellipse 0.4.3, ff 4.0.7, foreach 1.5.2, illuminaio 0.40.0, iterators 1.0.14, lattice 0.20-45, limma 3.54.0, mvtnorm 1.1-3, openssl 2.0.4, parallel 4.2.1, preprocessCore 1.60.0, restfulr 0.0.15, rjson 0.2.21, splines 4.2.1, tools 4.2.1, yaml 2.3.6, zlibbioc 1.44.0

References

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